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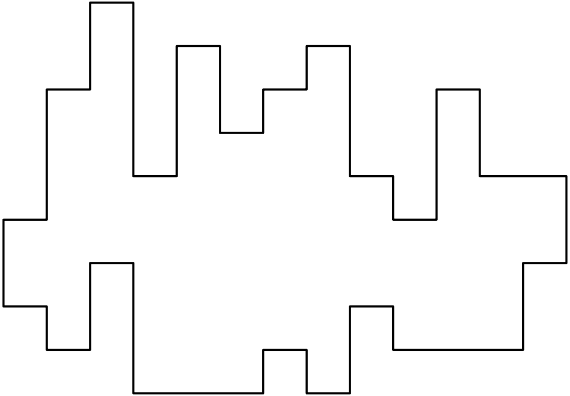
Computational Geometry and Applications Lab

Computing the **Straight Skeleton** of an **Orthogonal Monotone Polygon** in **Linear Time**

Günther Eder, Martin Held, and Peter Palfrader

Preliminaries

- P is an orthogonal x-monotone polygon with n vertices.



Preliminaries

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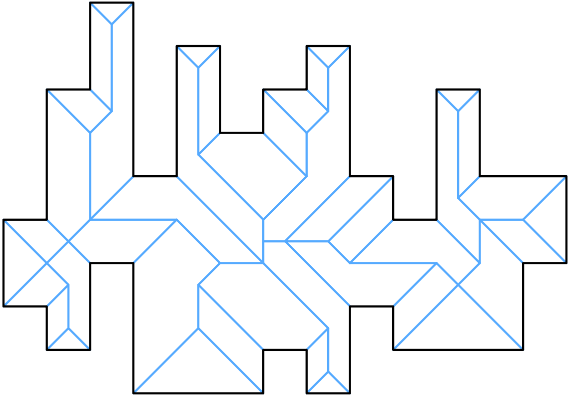
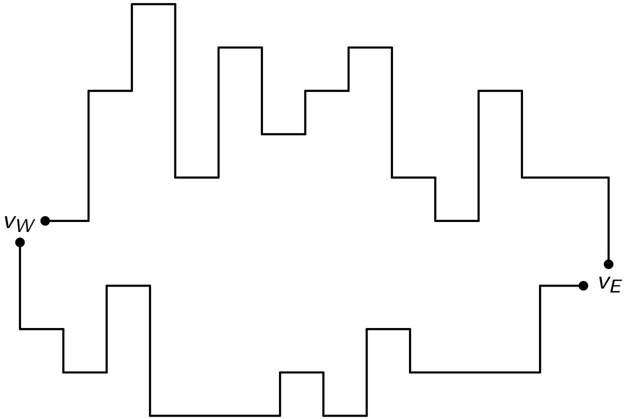
- P is an orthogonal x-monotone polygon with n vertices.
- $\mathcal{S}(P)$ denotes the straight skeleton of P.



Preliminaries

- P is an orthogonal x-monotone polygon with n vertices.
- $\mathcal{S}(P)$ denotes the straight skeleton of P.
- We split P into its upper and lower monotone chain.



Preliminaries

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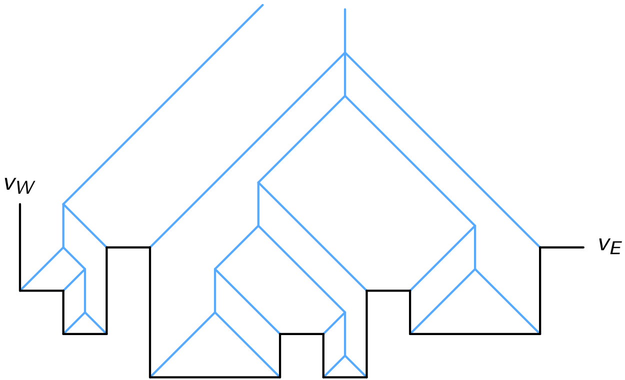
- P is an orthogonal x-monotone polygon with n vertices.
- $\mathcal{S}(P)$ denotes the straight skeleton of P.
- We split P into its upper and lower monotone chain.
- Looking at a single chain C, let $\mathcal{S}(C)$ denote its straight skeleton.



Algorithm Setup

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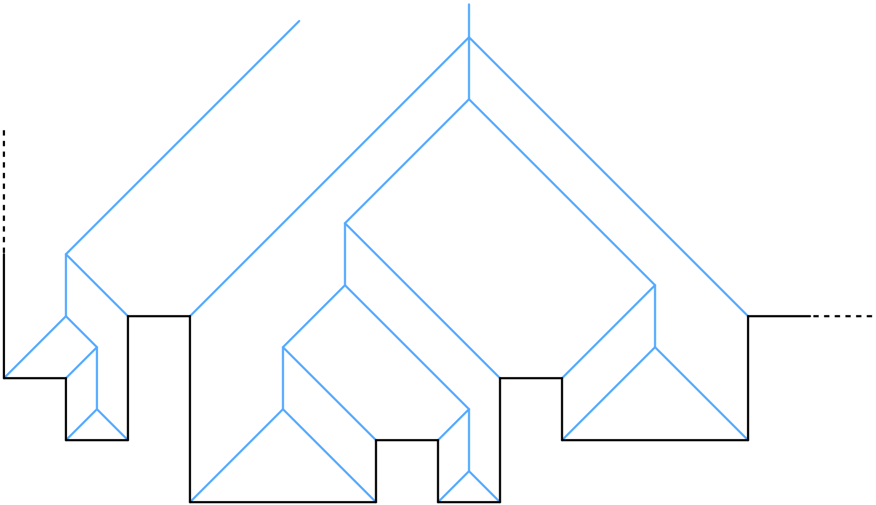
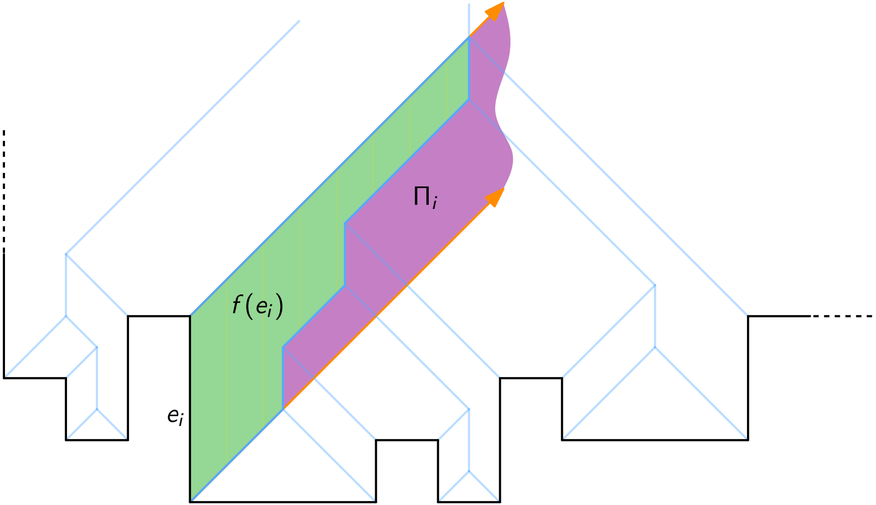
The arcs of $\mathcal{S}(C)$ have only three directions: $\binom{1}{1}$, $\binom{-1}{1}$, and $\binom{0}{1}$.



Algorithm Setup

A face $f(e_i)$ of $\mathcal{S}(C)$ lies inside of the half-plane slab Π_i.



Algorithm Setup

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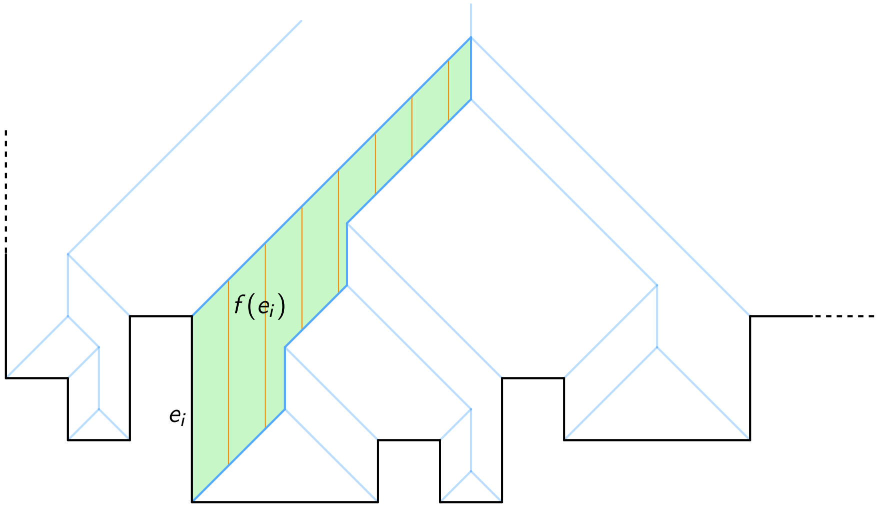
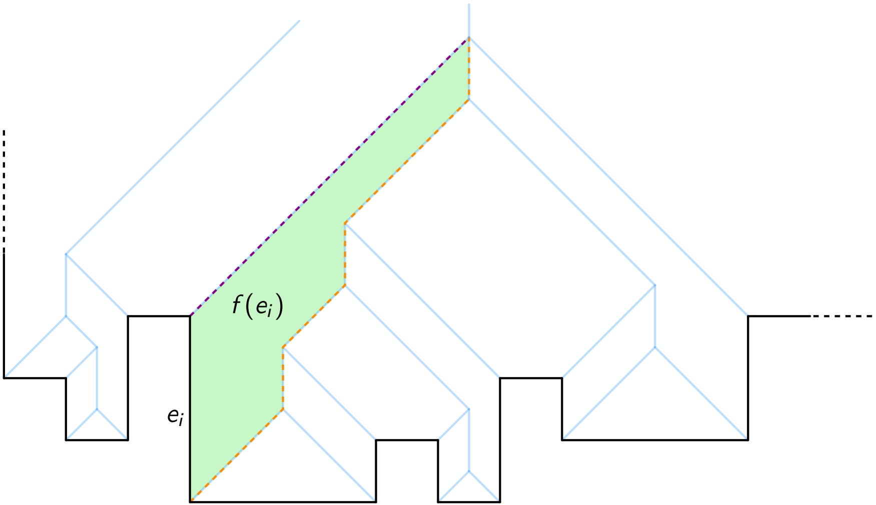
Also, $f(e_i)$ is monotone in respect its input edge as well as to a line perpendicular to it.



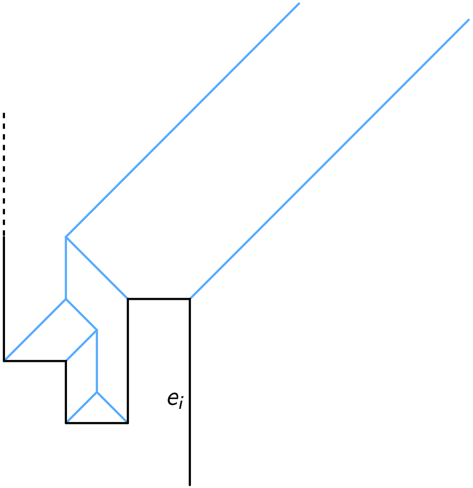
Algorithm Setup

Let us separate $f(e_i)$ into its left and right chain.



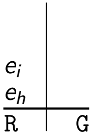
Algorithm Setup

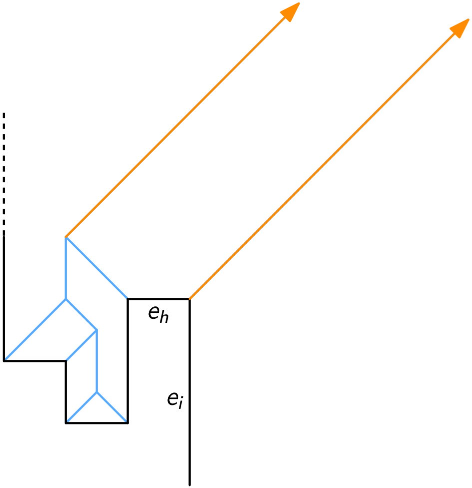
We maintain the partial straight skeleton $\mathcal{S}^*$ during our incremental construction.
It contains the left chains of all edges already inserted



Algorithm Setup

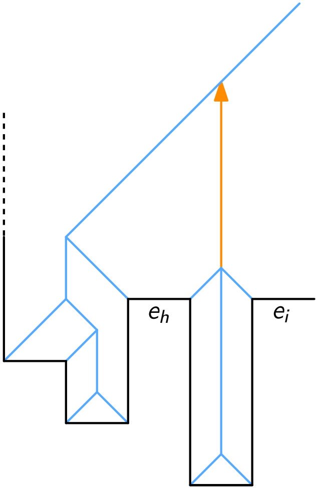
We maintain the partial straight skeleton $\mathcal{S}^*$ during our incremental construction.
It contains the left chains of all edges already inserted, as well as two stacks R





Algorithm Setup

We maintain the partial straight skeleton $\mathcal{S}^*$ during our incremental construction.
It contains the left chains of all edges already inserted, as well as two stacks R and G.



e_h	e_i
R	G

Constructing $\mathcal{S}(C)$

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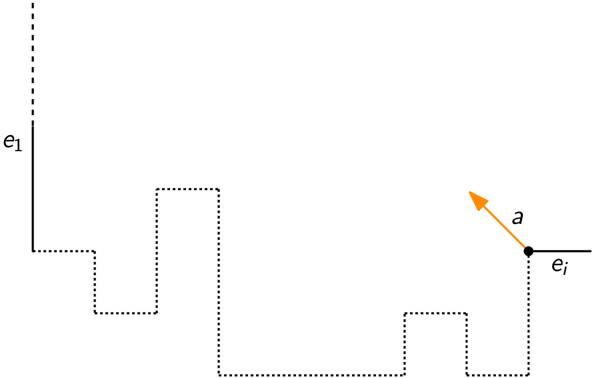
We start our incremental construction by adding e_1.

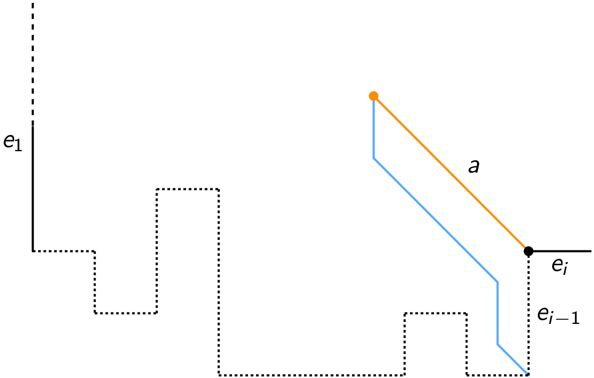
Constructing $\mathcal{S}(C)$

The first arc a of the left chain of $f(e_i)$ has $\binom{1}{1}$ or $\binom{-1}{1}$ direction.



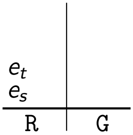
Constructing $\mathcal{S}(C)$

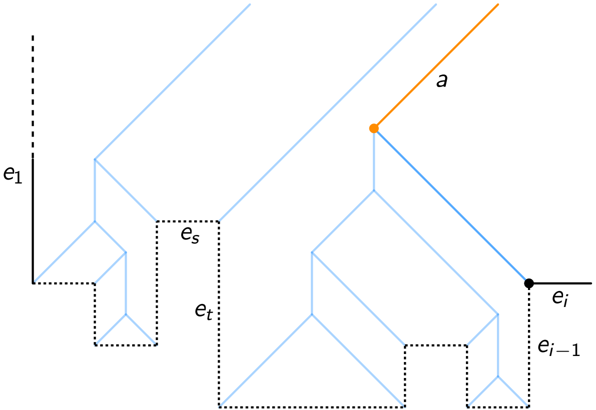
The first arc a of the left chain of $f(e_i)$ has $\binom{1}{1}$ or $\binom{-1}{1}$ direction.
It connects to the end of $f(e_{i-1})$'s left chain.



Constructing $\mathcal{S}(C)$

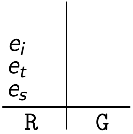
Subsequent arcs between e_i and the edge on top of R.

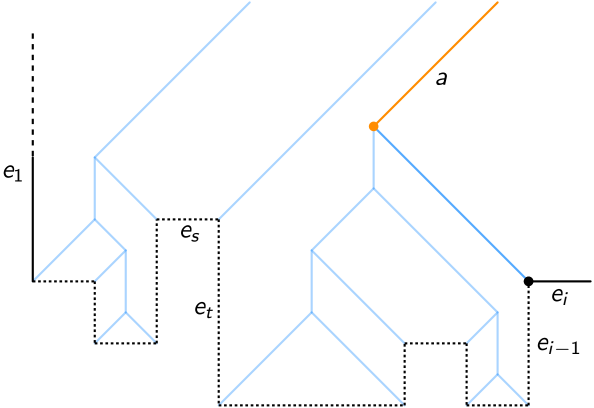




Constructing $\mathcal{S}(C)$

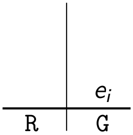
Subsequent arcs between e_i and the edge on top of R. The last arc of a chain ends in a ray,

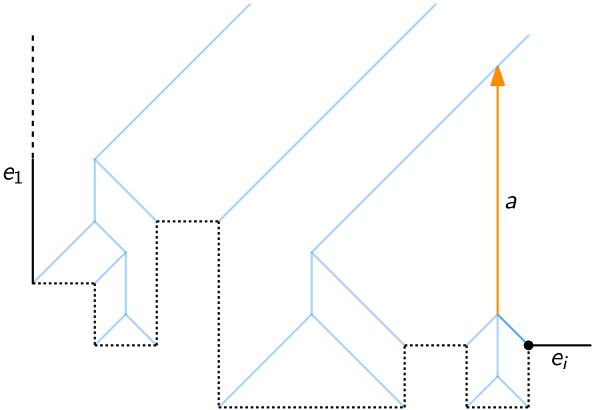




Constructing $\mathcal{S}(C)$

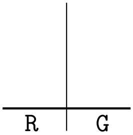
Subsequent arcs between e_i and the edge on top of R. The last arc of a chain ends in a ray, unfinished ghost arc,

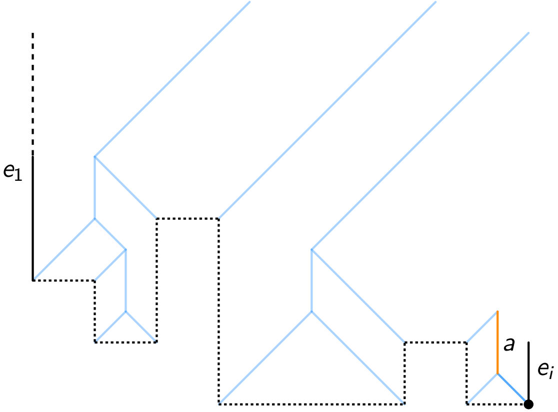




Constructing $\mathcal{S}(C)$

Subsequent arcs between e_i and the edge on top of R. The last arc of a chain ends in a ray, unfinished ghost arc, or bounded vertical arc.





Arc a has $\binom{1}{1}$ Direction

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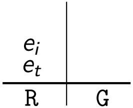
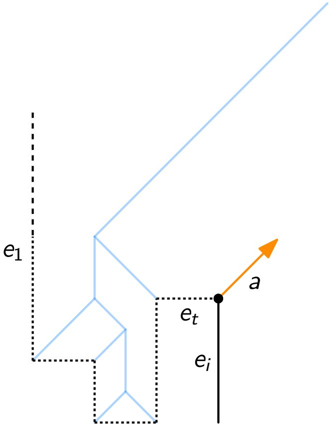
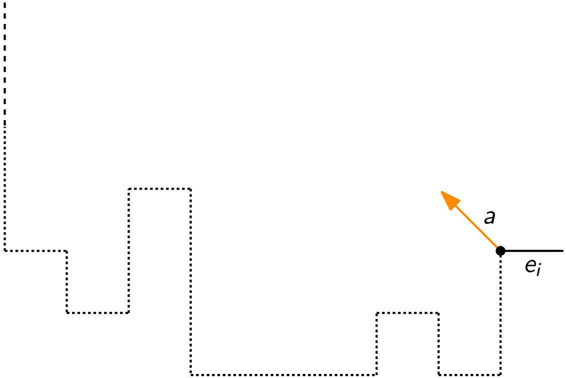
We follow with a case distinction for the next arc a added in the left chain of e_i.
Arc a is a ray and we push e_i onto R.





Arc a has $\binom{-1}{1}$ Direction

Arc a is either a bounded arc or a ray.



Arc a has $\binom{-1}{1}$ Direction

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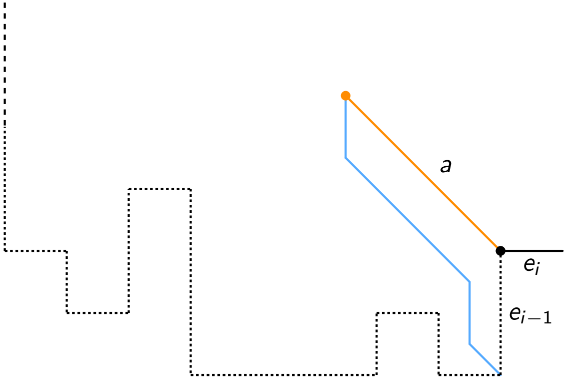
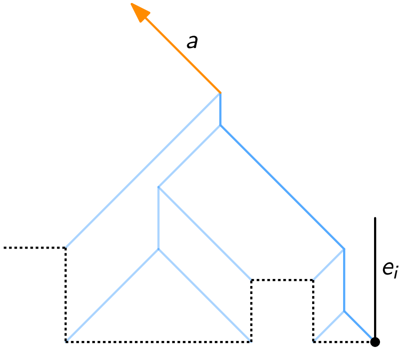
If the left chain of e_{i-1} terminates in a bounded arc, and a is the first arc on the left chain of e_i, it ends where the left chain of e_{i-1} ends.



Arc a has $\binom{-1}{1}$ Direction

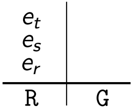
Otherwise, we look at e_t at the top of R. If e_t does not terminate in a $\binom{1}{1}$ ray, a is a $\binom{-1}{1}$ ray, e_i is pushed onto R, and the chain is completed.

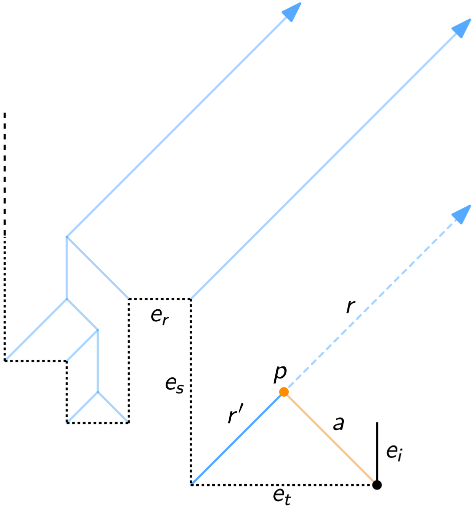
R	G
e_i	



Arc a has $\binom{-1}{1}$ Direction

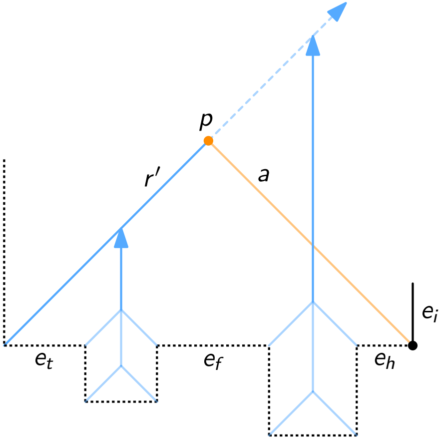
Otherwise, the left chain of e_t terminates in a $\binom{1}{1}$ ray r. At p arc a intersects ray r. In $f(e_{i-1})$ we modify r into a bounded arc r' that ends at p, where a ends as well.





Arc a has $\binom{-1}{1}$ Direction

Finally we have to process the elements of G below r' and a.



R	G
e_t	e_h
	e_f

Arc a has $\binom{-1}{1}$ Direction

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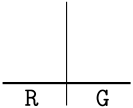
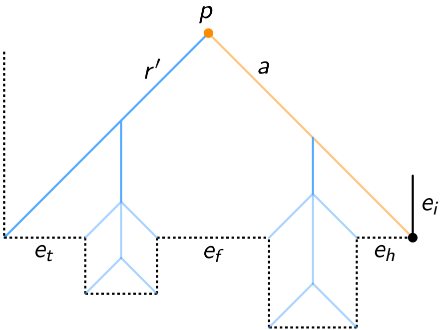
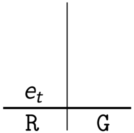
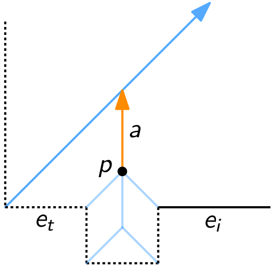




Arc a has $\binom{0}{1}$ Direction

Arc a is either a ghost arc or bounded vertical arc, starting at a point p.

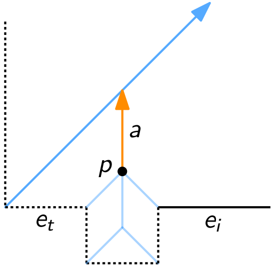




Arc a has $\binom{0}{1}$ Direction

Arc a is either a ghost arc or bounded vertical arc, starting at a point p. In case a is a ghost arc we push e_i onto `G`.

e_t	e_i
`R`	`G`



Arc a has $\binom{0}{1}$ Direction

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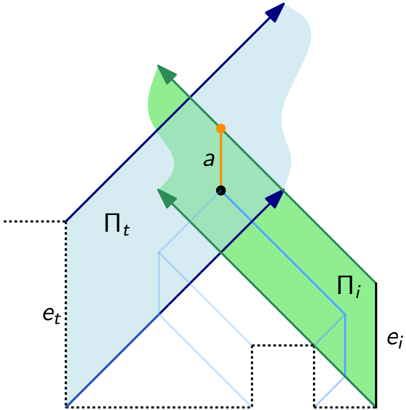
Otherwise, a is the line segment from p that is contained in both Π_t and Π_i.



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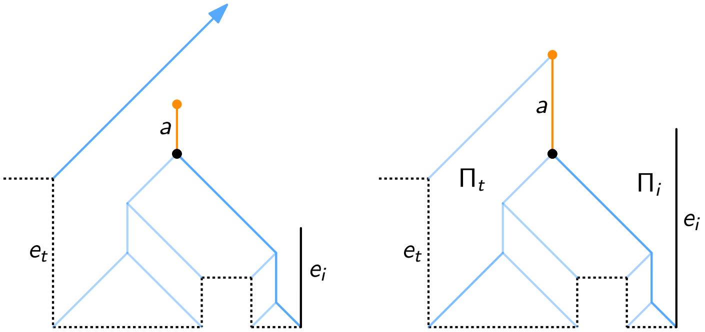

Finalizing $\mathcal{S}(C)$

- We process the elements that remain on `G`.

Finalizing $\mathcal{S}(C)$

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- We process the elements that remain on `G`.
- All arcs inserted intersect only rays or ghost arcs.

Finalizing $\mathcal{S}(C)$

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- We process the elements that remain on `G`.
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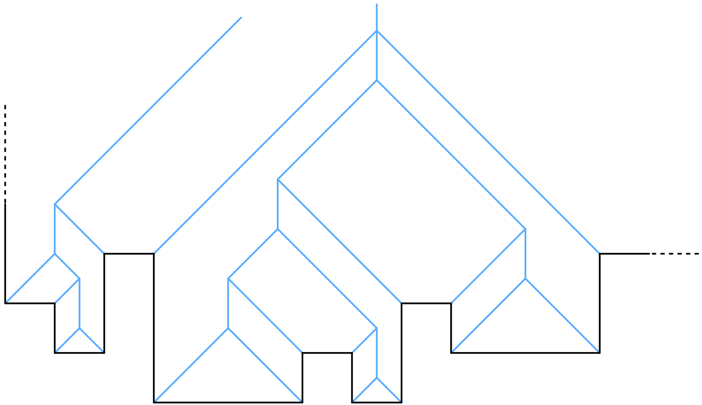
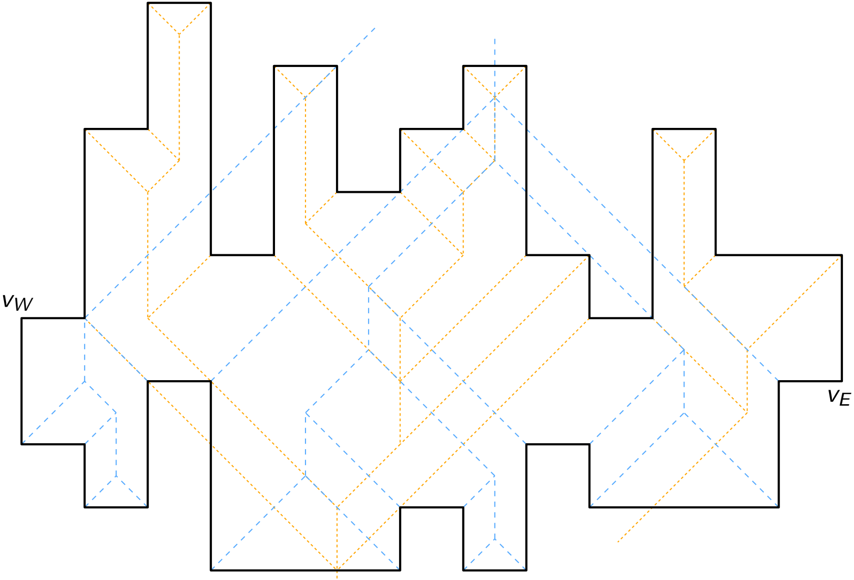
Theorem

Our incremental construction approach creates $\mathcal{S}(C)$ in $\mathcal{O}(n)$ time.



Skeleton Merging



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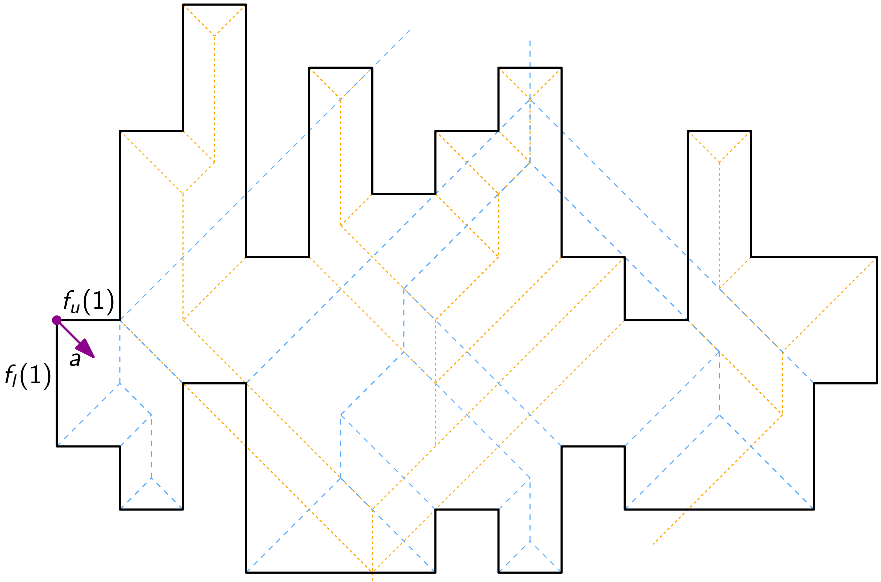

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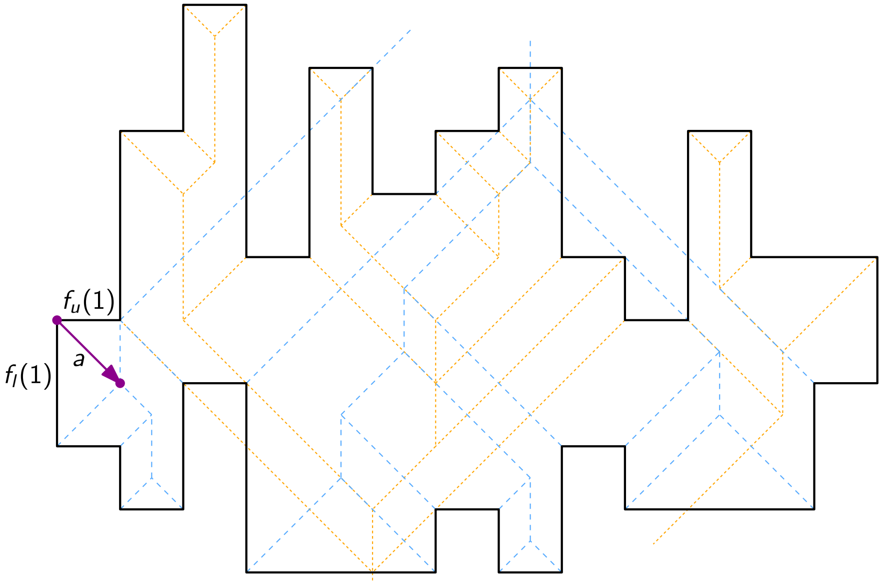

Skeleton Merging

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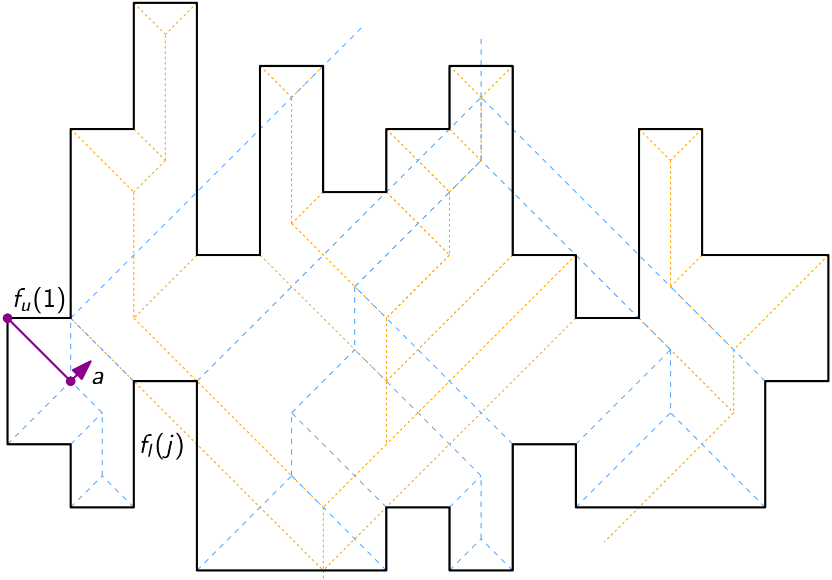

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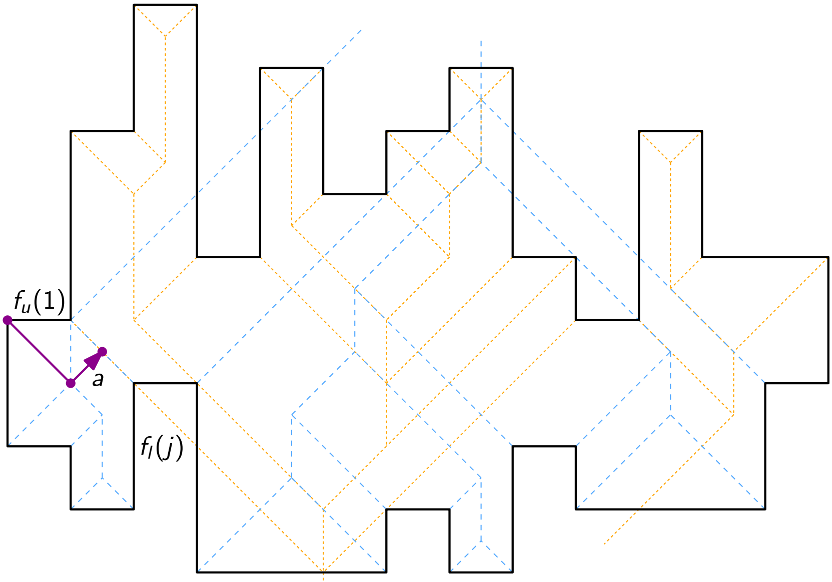

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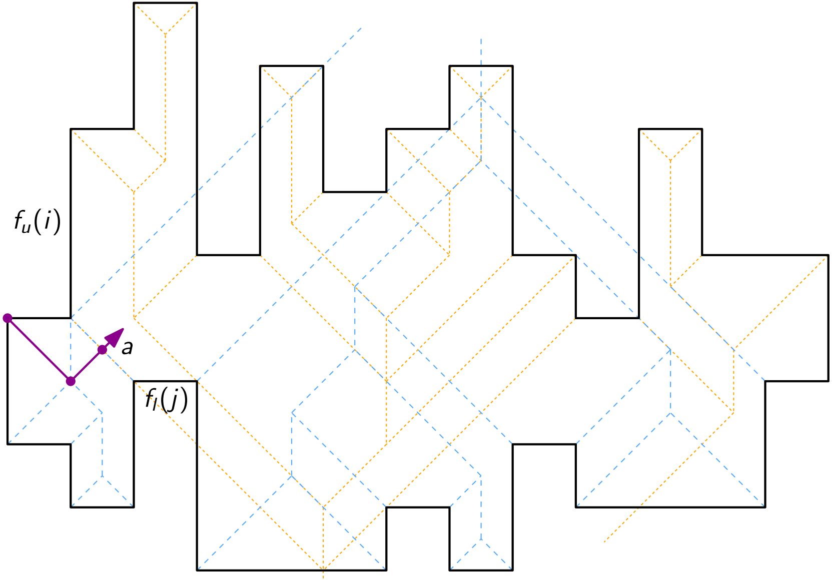

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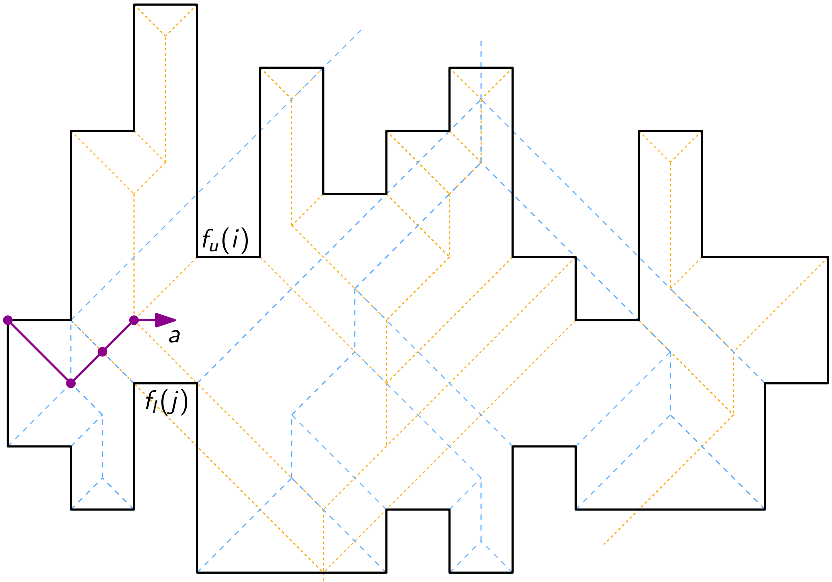

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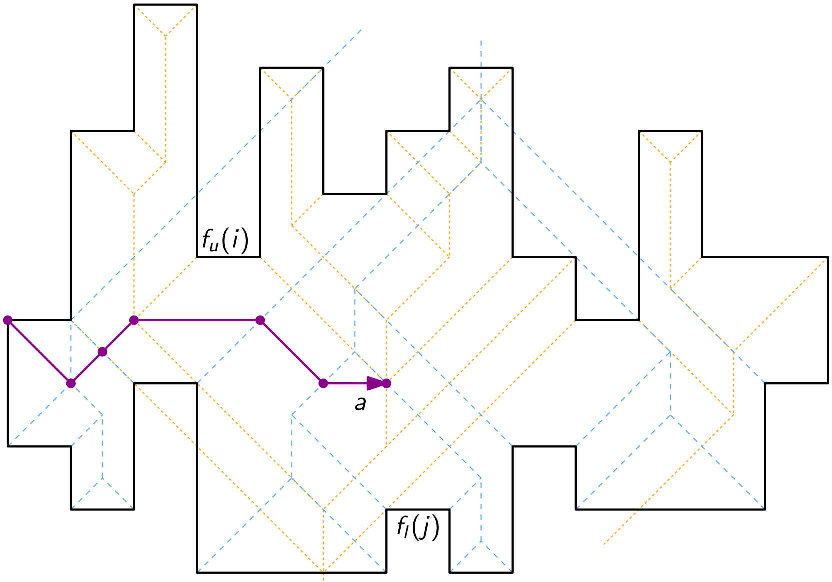

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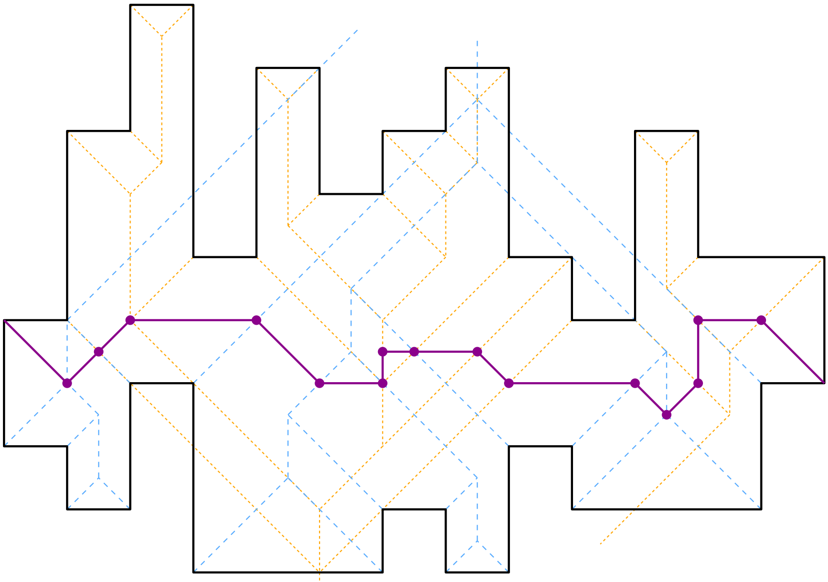

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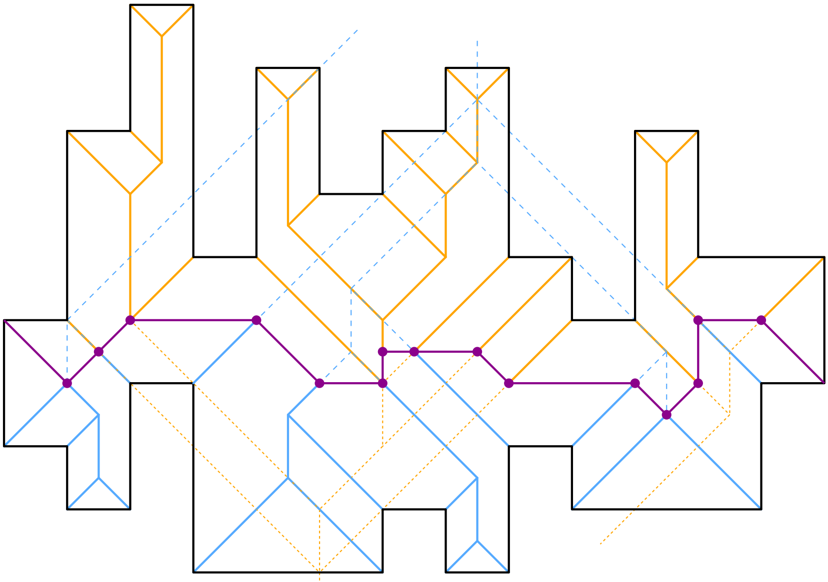

Skeleton Merging

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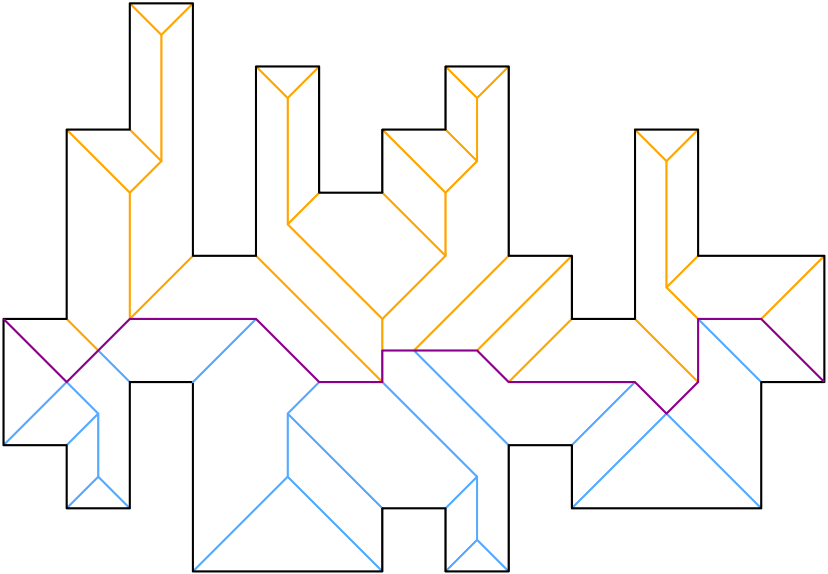

Summary

- Incremental construction of $\mathcal{S}(C)$ in linear time.
- Merge of both straight skeletons in linear time.

Questions?