# Computing Positively Weighted Straight Skeletons of Simple Polygons Using an Induced Line Arrangement

Günther Eder and Martin Held

University of Salzburg
FB Computerwissenschaften
5020 Salzburg, Austria

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# Related Work & Overview

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- The straight skeleton was introduced by Aichholzer et al. 1995 [1].
- Eppstein and Erickson [3] introduced in 1999 an algorithm with the current best worst-case complexity: For a simple polygon (with holes) it requires $\mathcal{O}(n^{17/11+\varepsilon})$ time and space to compute the (weighted) straight skeleton.
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- Our work is based on the work of Huber and Held (IJCGA 2012) on straight-skeleton computation based on motorcycle graphs [4].
- Using an *extended wavefront* they transform split events into edge events, shifting the complexity to another event.
- We revisit their work and show required changes to apply their approach to the weighted scenario.

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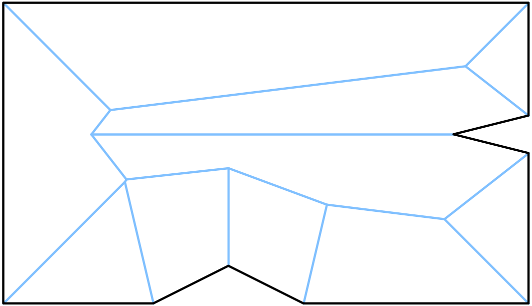
- Consists only of straight line segments.
- Defined by a propagation process:
  - Edges move inwards in a parallel manner at unit speed.
  - The vertices of the *wavefront polygons* trace out *arcs*.
  - Two events: edge event and split event



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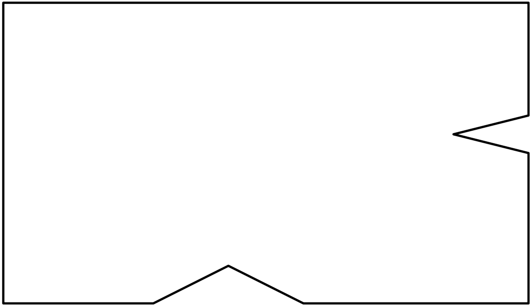

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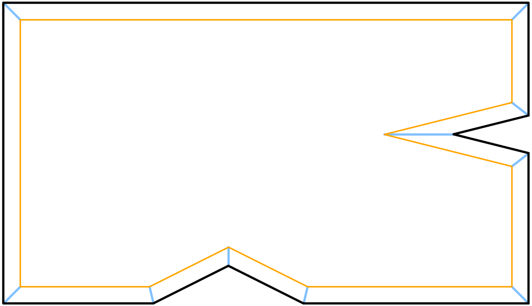

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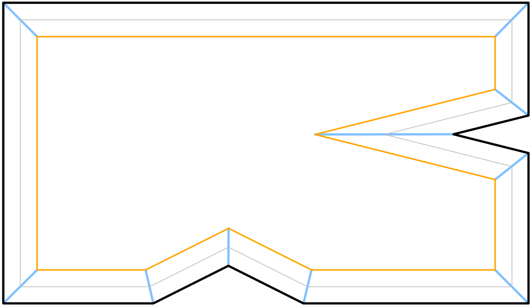

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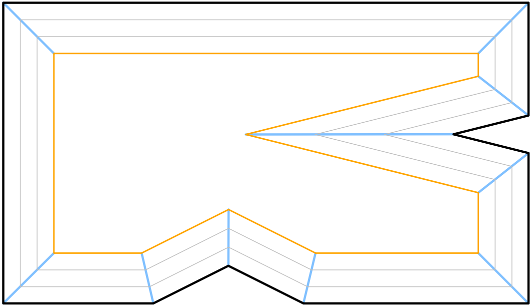

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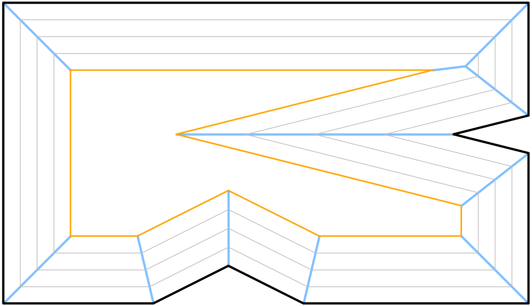

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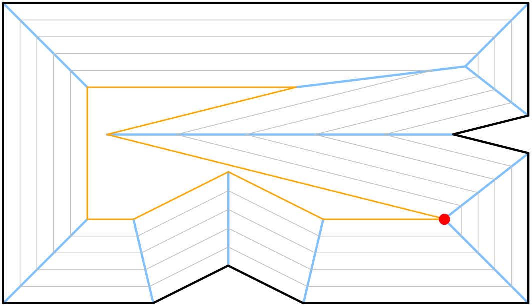

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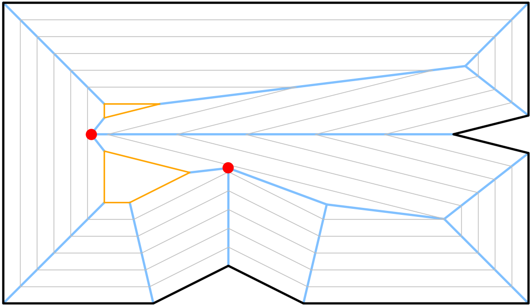

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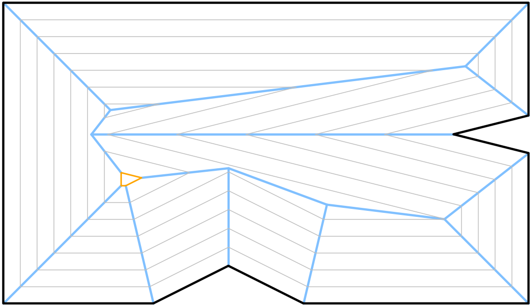

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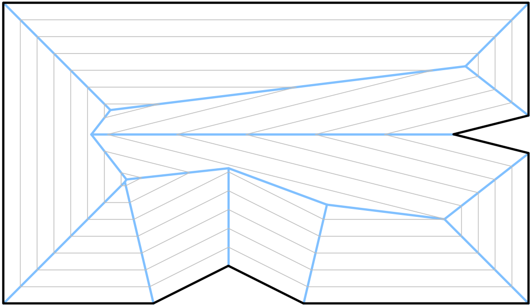

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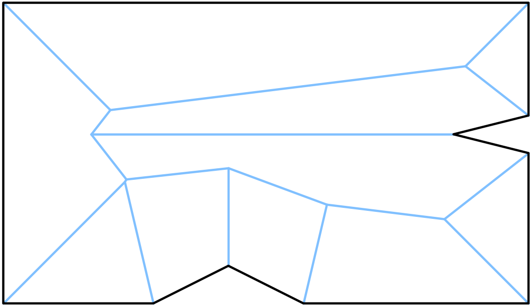
  - The *straight skeleton* consists of the blue arcs.



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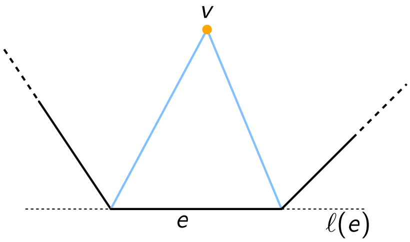
**Edge Events**

- Compute vanishing time for every edge if finite.
- Enqueue all events in priority queue.
- $\mathcal{O}(n \log n)$ time and $\mathcal{O}(n)$ space.



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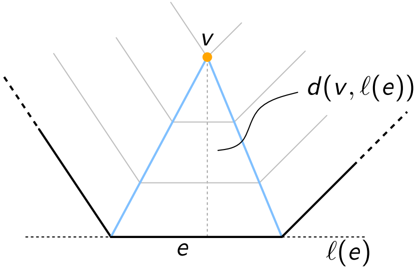

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- Let $v$ be a reflex vertex of $P$. Then there are $\mathcal{O}(n)$ possible split events for $v(t)$ in $\mathcal{W}_P(t)$.
- Enqueue event $v'$ closest to $v$ in $\mathcal{O}(n + \log n)$ time.
- If $v'$ is not reached again $\mathcal{O}(n + \log n)$ for $v''$.
- We may miss $\mathcal{O}(n)$ times.
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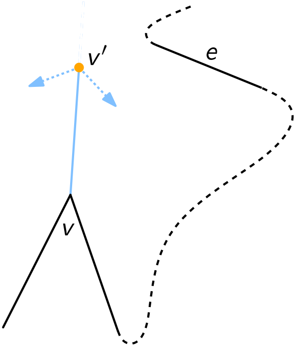

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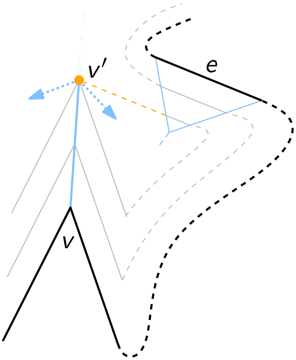

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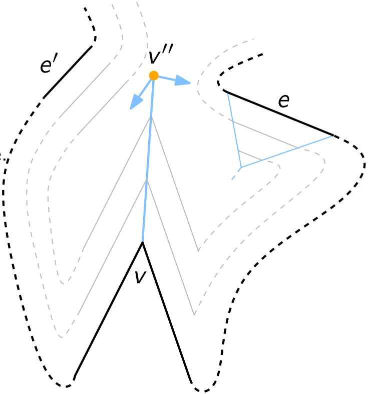

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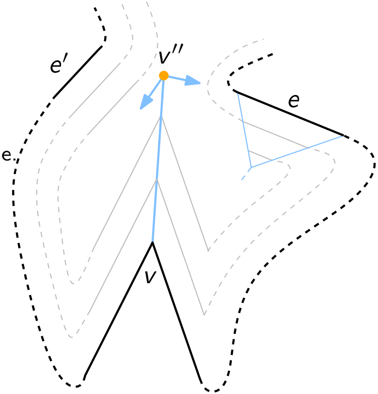

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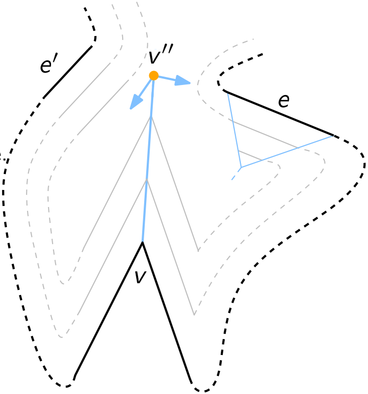


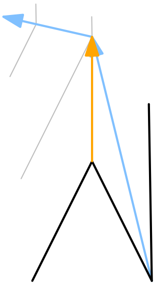

**Multi-Split Event**

- When two reflex wavefront vertices meet at a common point[2].

[2] Related to vertex event [3].

# Tracing Reflex Arcs

- Every event, where a reflex wavefront vertex is involved, reduces the number of reflex vertices in $\mathcal{W}_P(t)$.
- *Can we know these reflex arcs in advance?*



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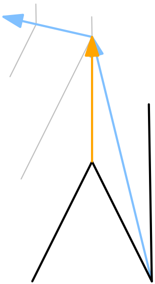

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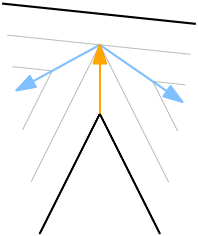

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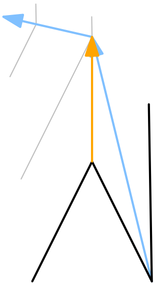

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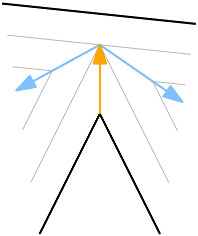

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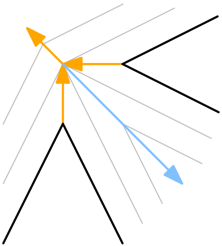

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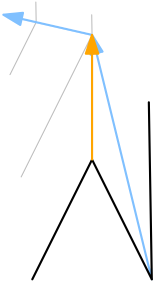

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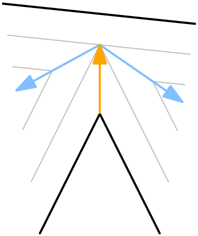

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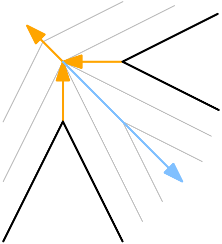

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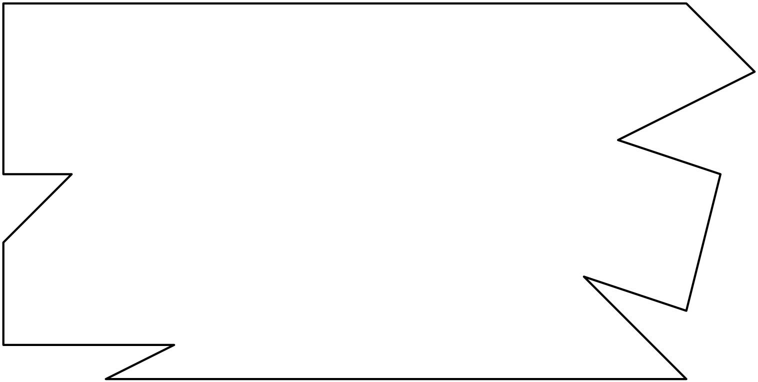
- Start a motorcycle $m$ for every reflex vertex $v$ of $\mathcal{W}_P(t)$ such that $m$ inherits the velocity vector of $v$.
- Every motorcycle leaves a trace behind and stops if it crashes into another trace or the polygon boundary.
- The *motorcycle graph* $\mathcal{M}(P)$[3] is formed when all motorcycles have crashed.



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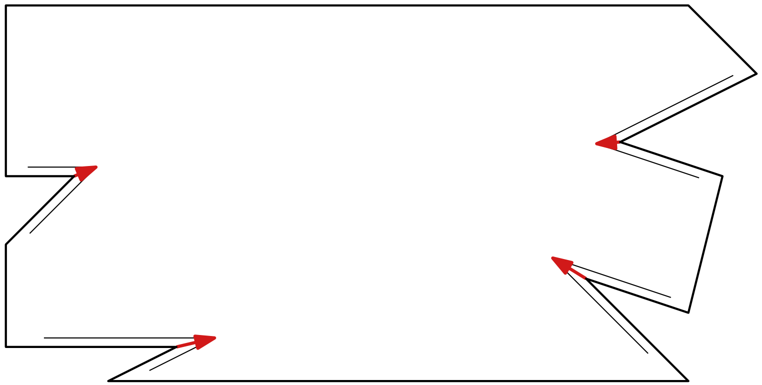

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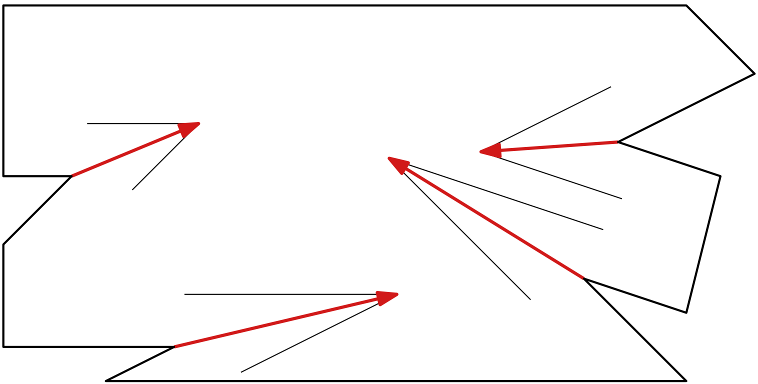

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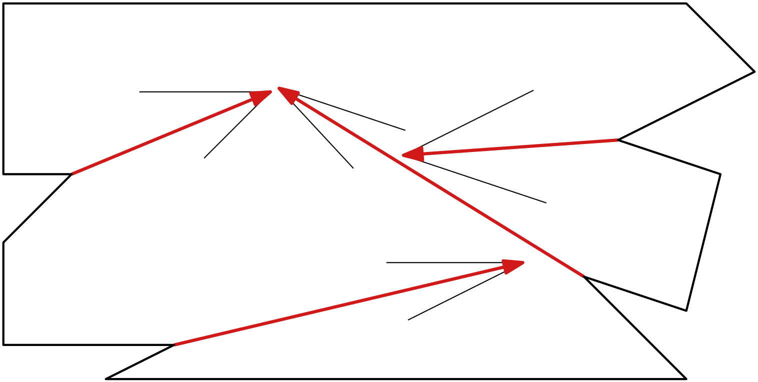

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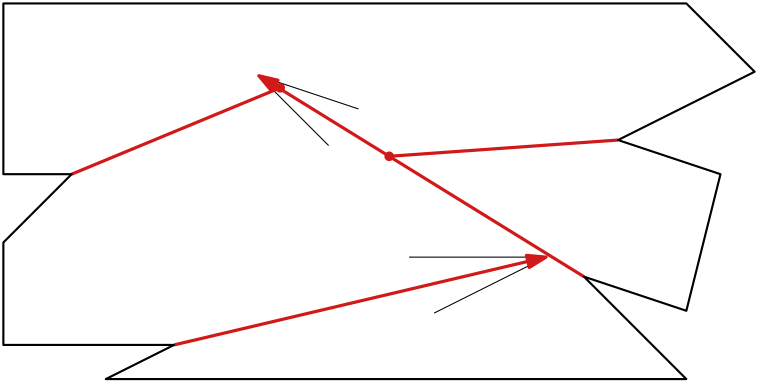

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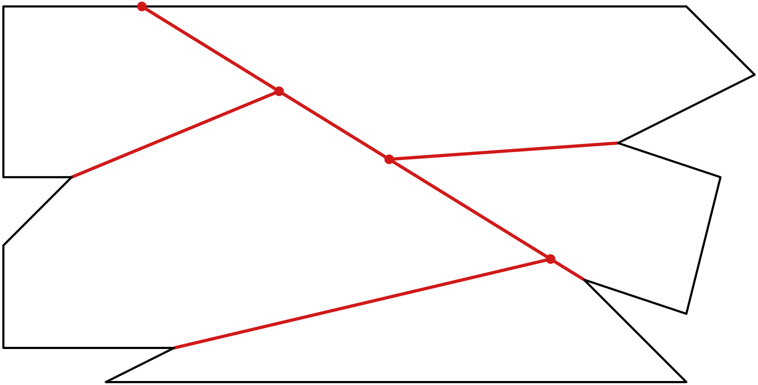

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# Motorcycle Graph and $\mathcal{S}(P)$ (cont'd)

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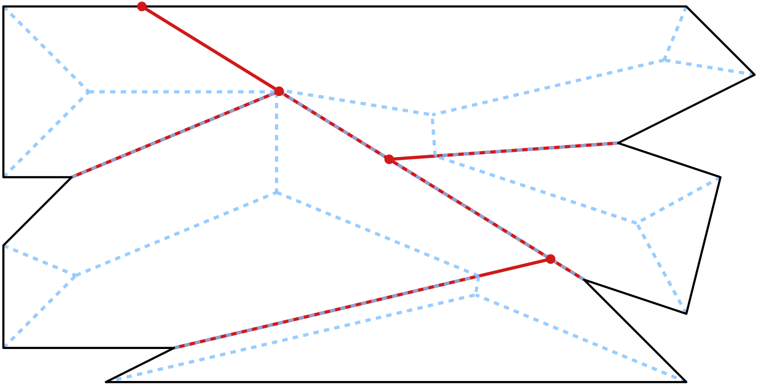

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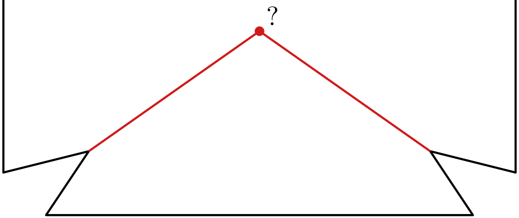


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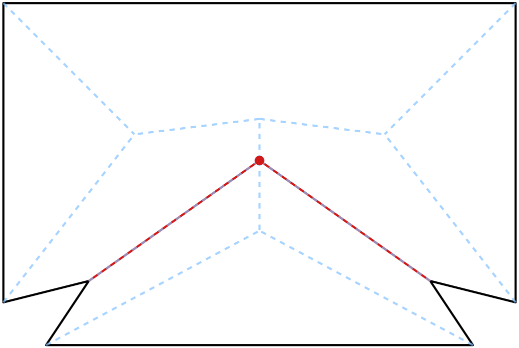

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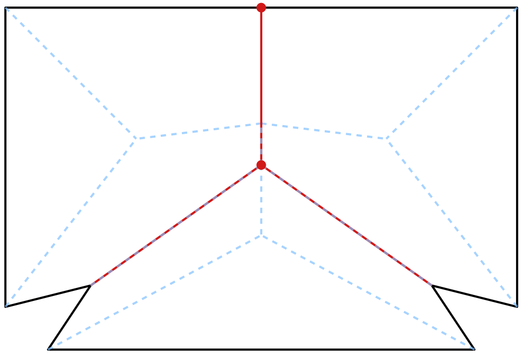

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## Extended Wavefront $\mathcal{W}_P^*$

- Huber and Held introduce the *extended* wavefront $\mathcal{W}_P^*(t)$.
- $\mathcal{W}_P^*(t)$ consists of the wavefront at time $t$ and the portion of $\mathcal{M}(P)$ that lies inside that wavefront.
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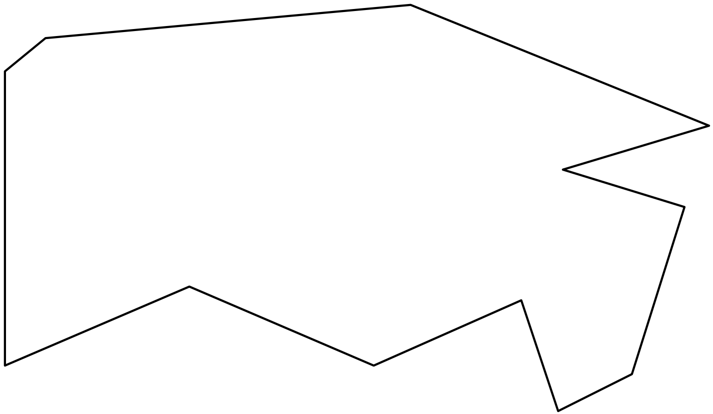

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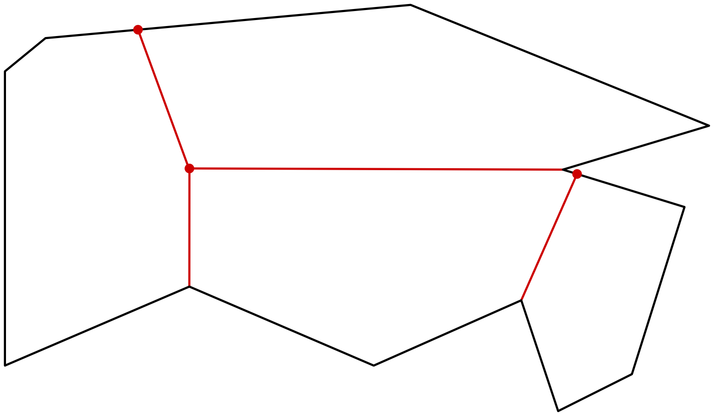

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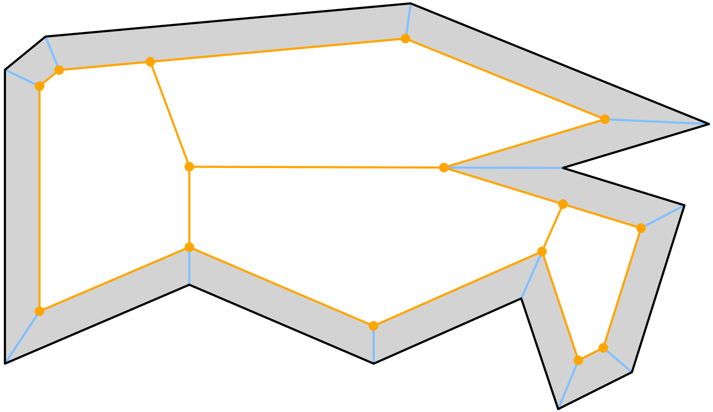

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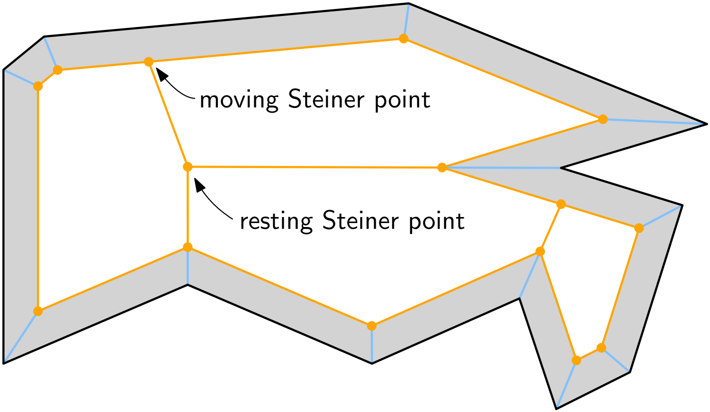

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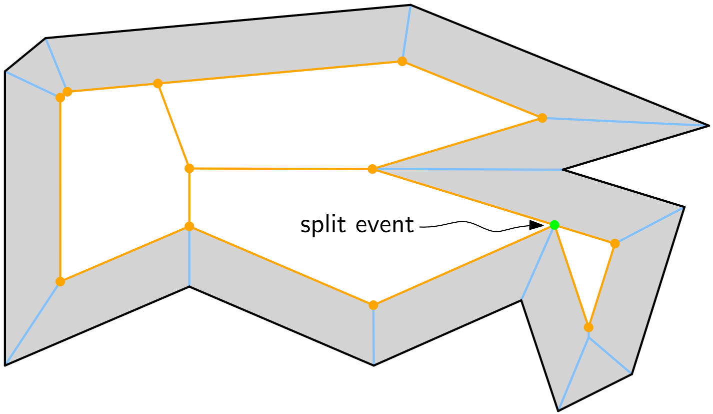

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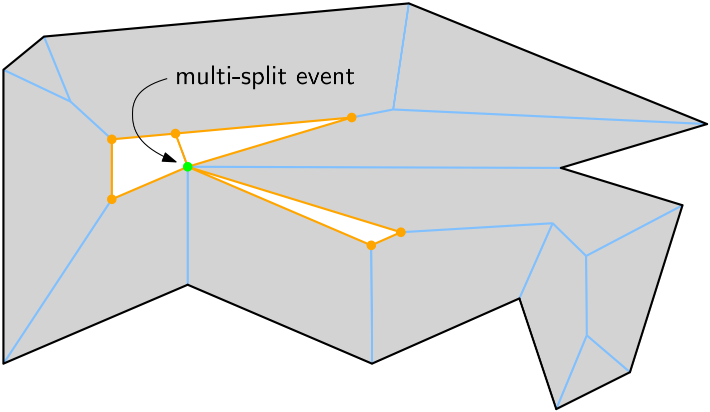

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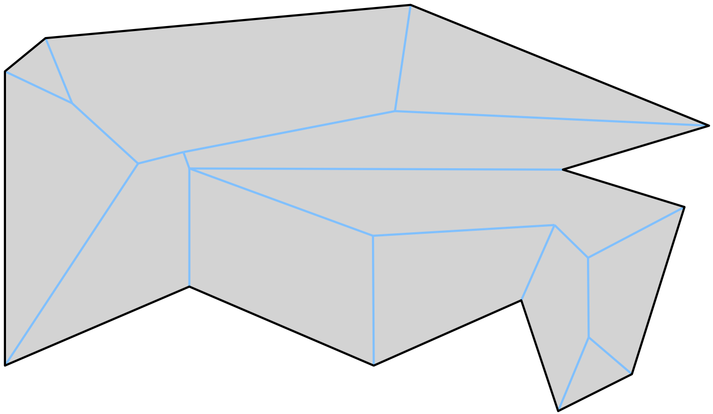

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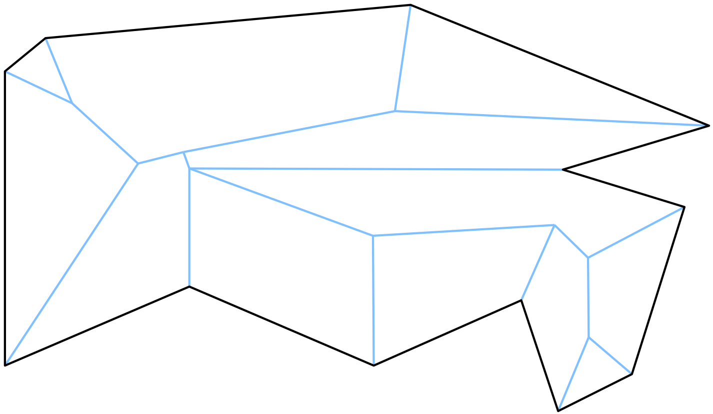

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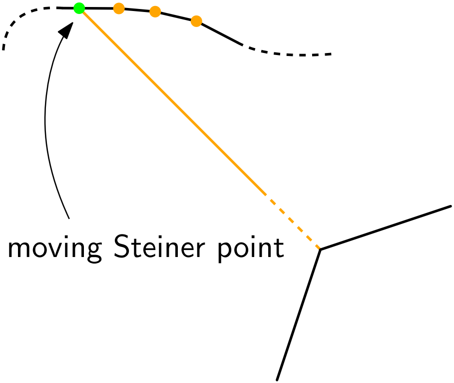

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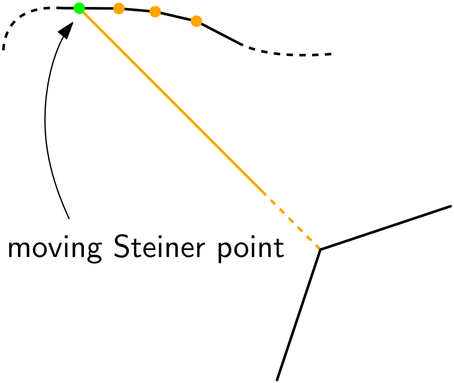

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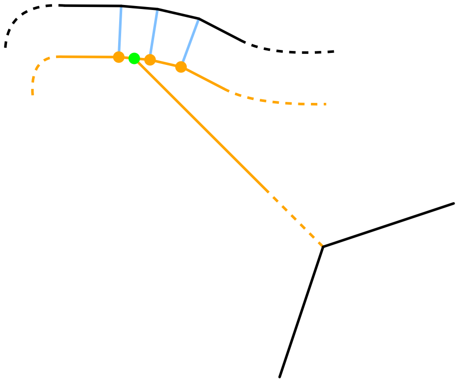

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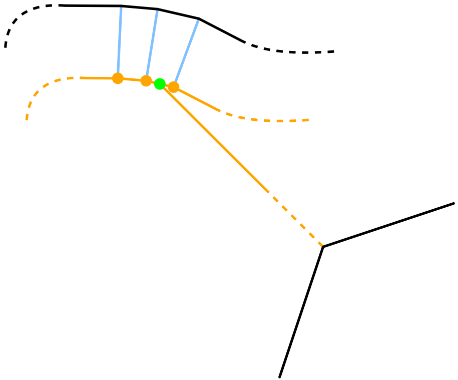

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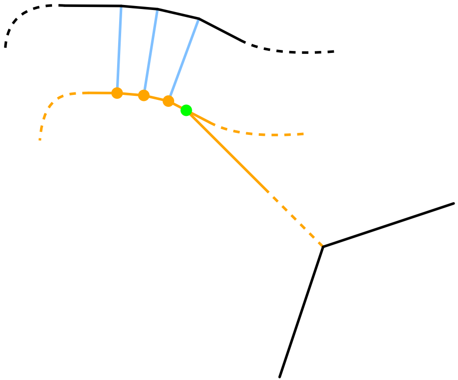
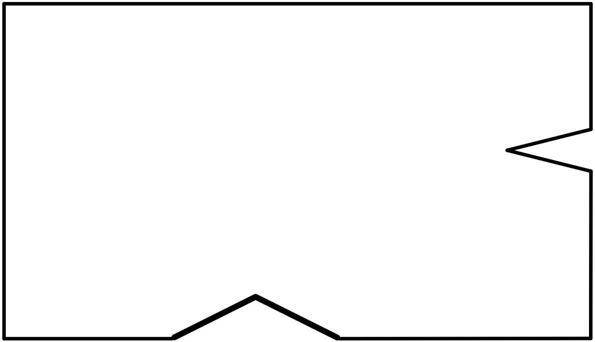


# Weighted Straight Skeleton $\mathcal{S}(P, \sigma)$

- Wavefront $\mathcal{W}_P(t, \sigma)$ traces out the weighted straight skeleton $\mathcal{S}(P, \sigma)$, s.t. the weight function $\sigma \in \mathbb{R}^+$ provides the strictly positive edge weights.
- Thick edges have $\sigma$ of about 3, unit weight otherwise.
- $\mathcal{S}(P)$ (left) and $\mathcal{S}(P, \sigma)$ (right).



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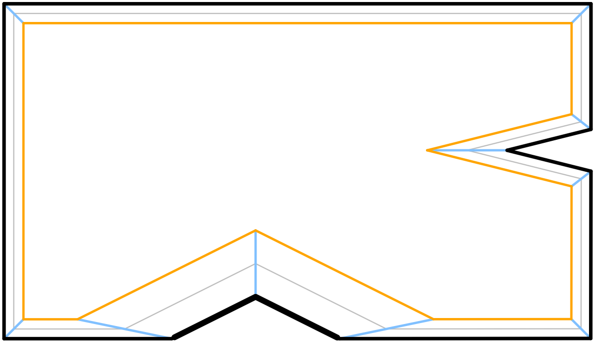

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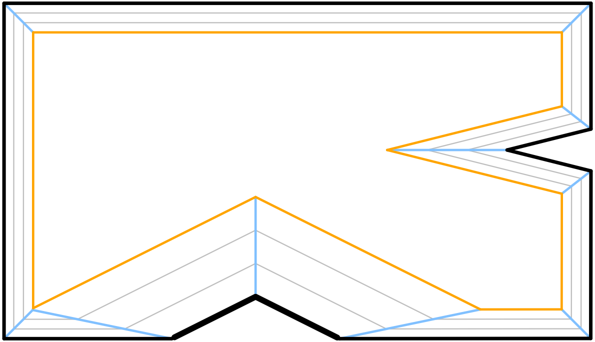

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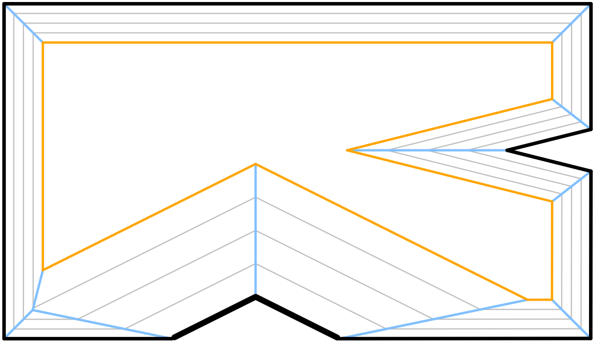

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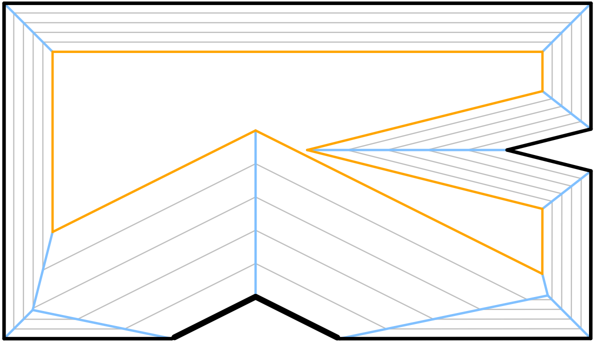

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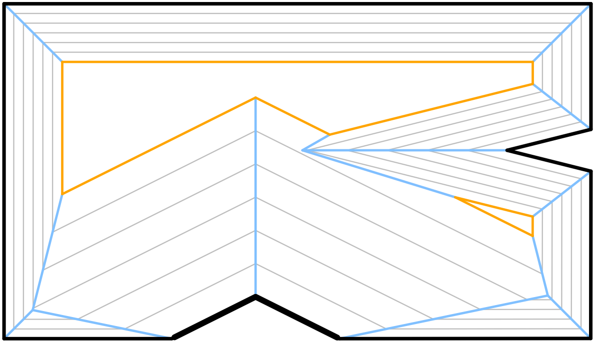

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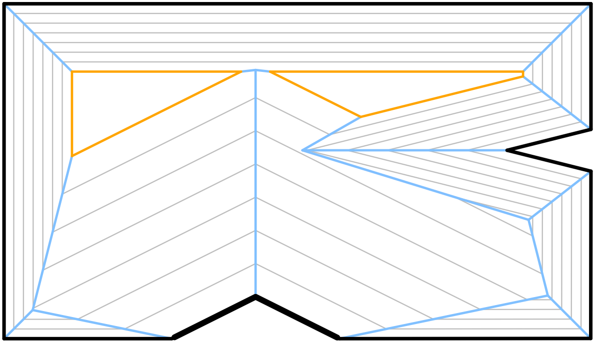

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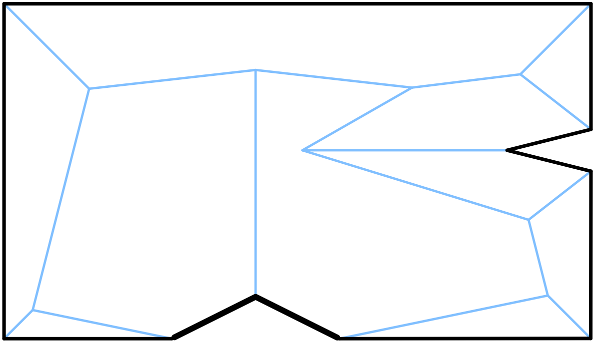

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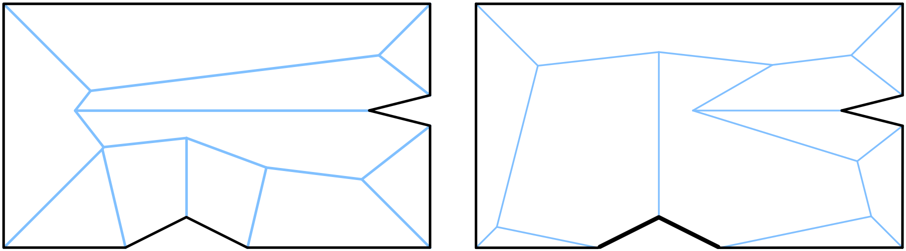

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- Reflex vertices of $\mathcal{W}(P,\sigma)$ are not limited by first event.
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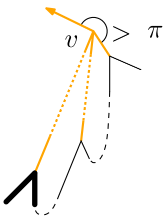


mutli-split event

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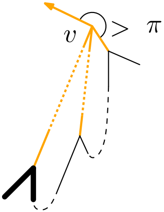


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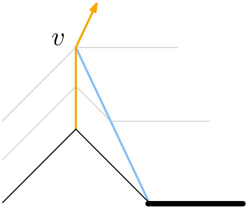


edge event

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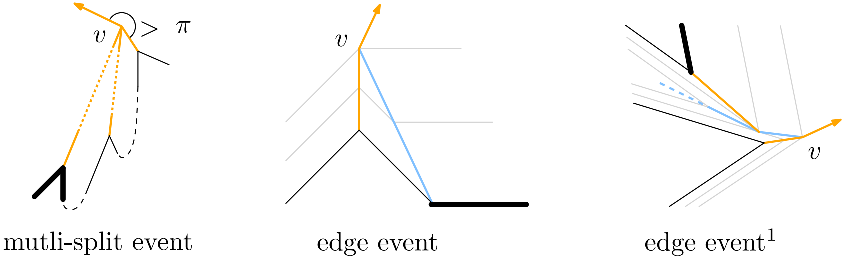


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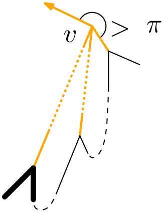


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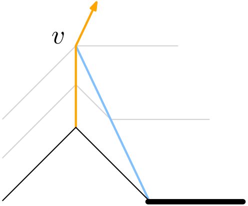
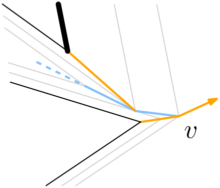



edge event

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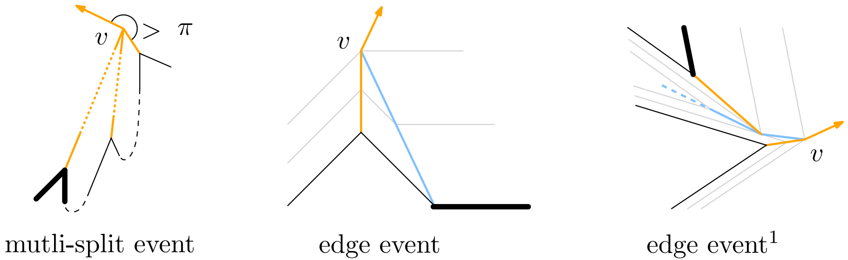


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# Induced Line Arrangement $\mathcal{A}(P)$

- For every reflex vertex $v$ of $P$ we construct a line segment $s$.
- Let $s$ start at $v$, lie on $v(t)$ of $\mathcal{W}_P(t, \sigma)$, and end at the first intersection with the boundary of $P$.
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**Linear or Quadratic Space**

- Store all $\mathcal{O}(r^2)$ intersections in a sorted manner: $\mathcal{O}(r^2 \log r)$ time and $\mathcal{O}(r^2)$ space. Obtain the next intersection in $\mathcal{O}(1)$ time.
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**Space Time Trade-off**

- For a fixed $k$ in $1 \leq k \leq r$. Let $s$ a segement in $\mathcal{A}(P)$.
- We compute and store the next $k$ intersections on $s$ in $\mathcal{O}(r \log r)$ time.
- On $s$ are at most $r$ intersections points, thus we have to compute the next $k$ intersections at most $r/k$ times.
- We require $\mathcal{O}(kr)$ space for $\mathcal{A}(P)$.

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- The set of all such line segments forms $\mathcal{A}(P)$.

**Linear or Quadratic Space**

- Store all $\mathcal{O}(r^2)$ intersections in a sorted manner: $\mathcal{O}(r^2 \log r)$ time and $\mathcal{O}(r^2)$ space. Obtain the next intersection in $\mathcal{O}(1)$ time.
- Store only the closest intersection to a point for all segments in $\mathcal{A}(P)$, i.e., overall $\mathcal{O}(r)$ intersections. Obtain the next intersection in $\mathcal{O}(r)$ time.

**Space Time Trade-off**

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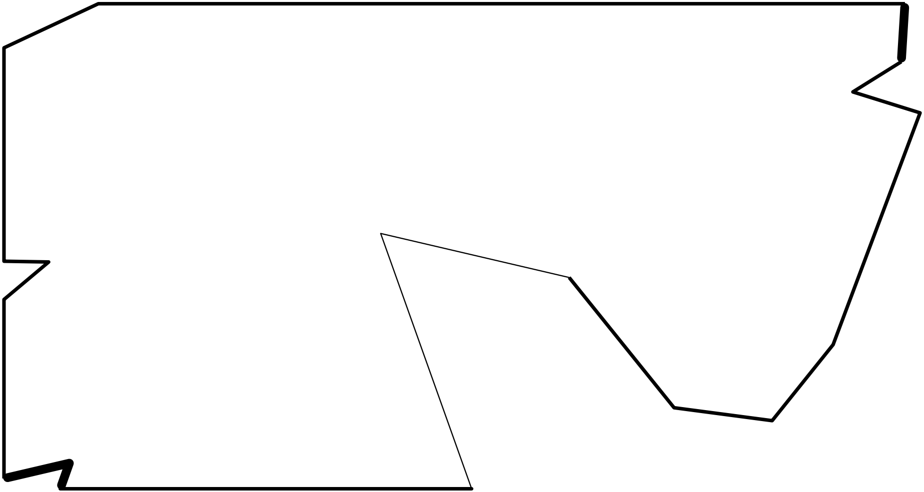

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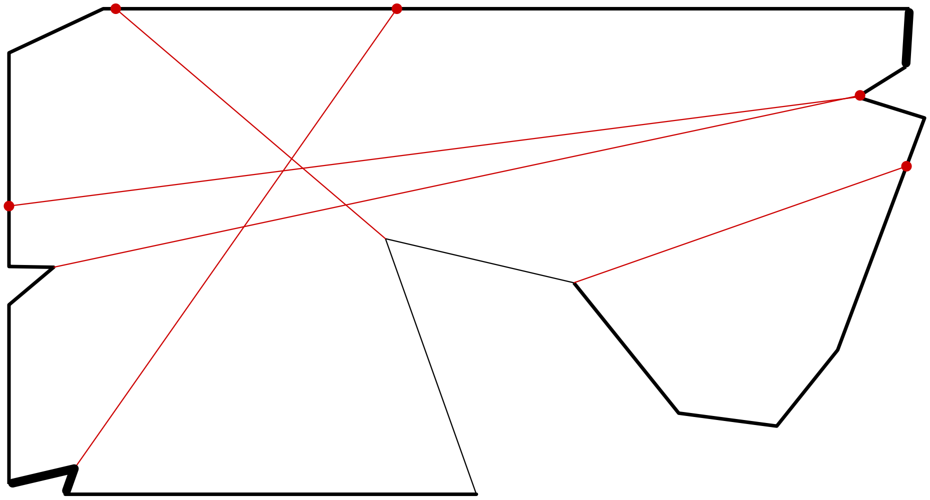

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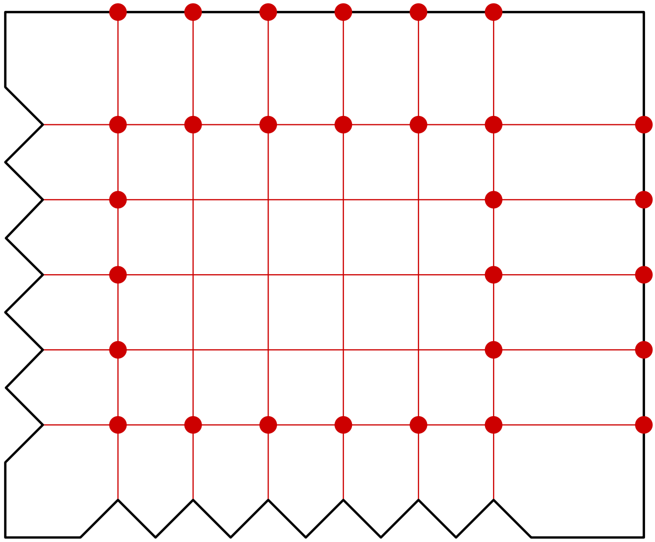

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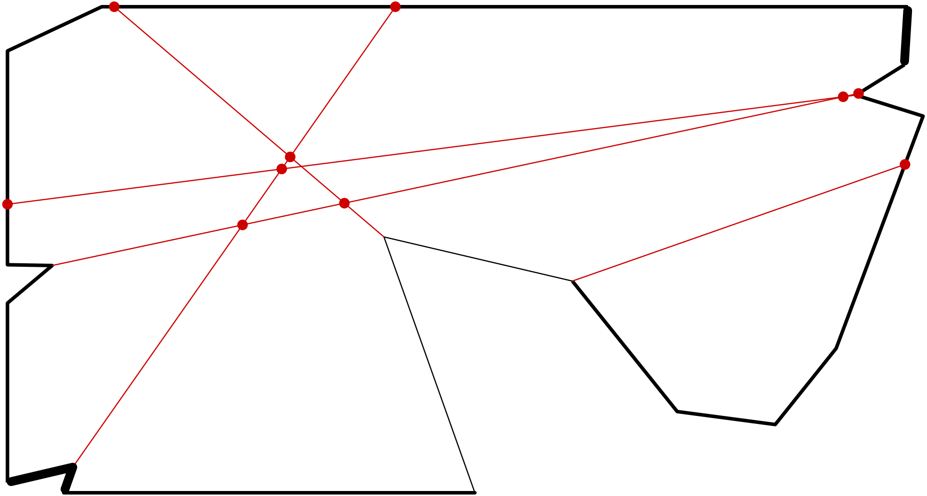

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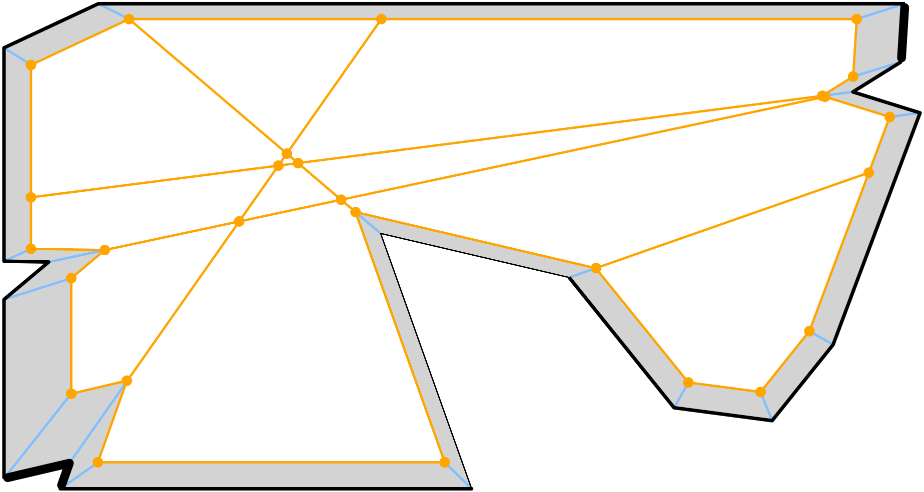

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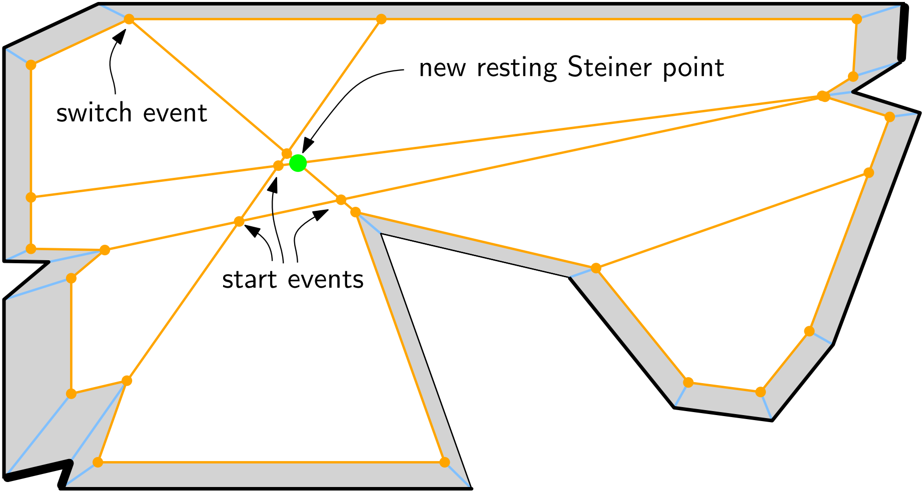

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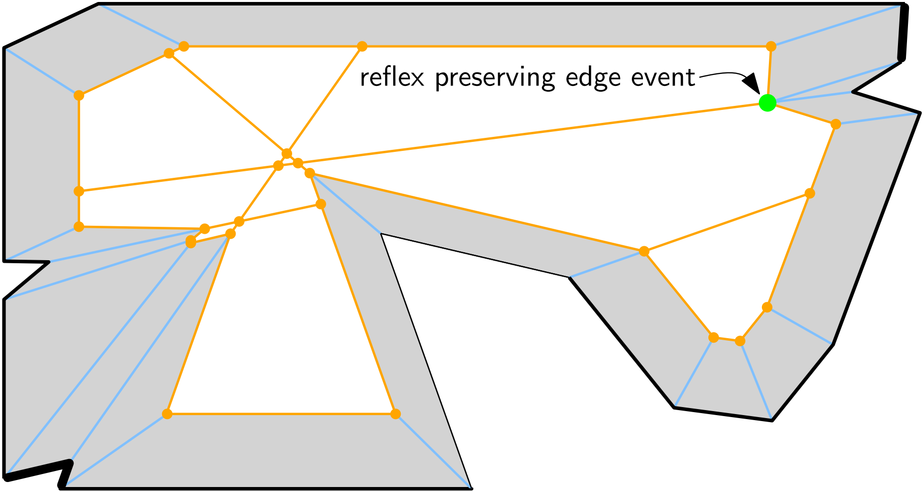

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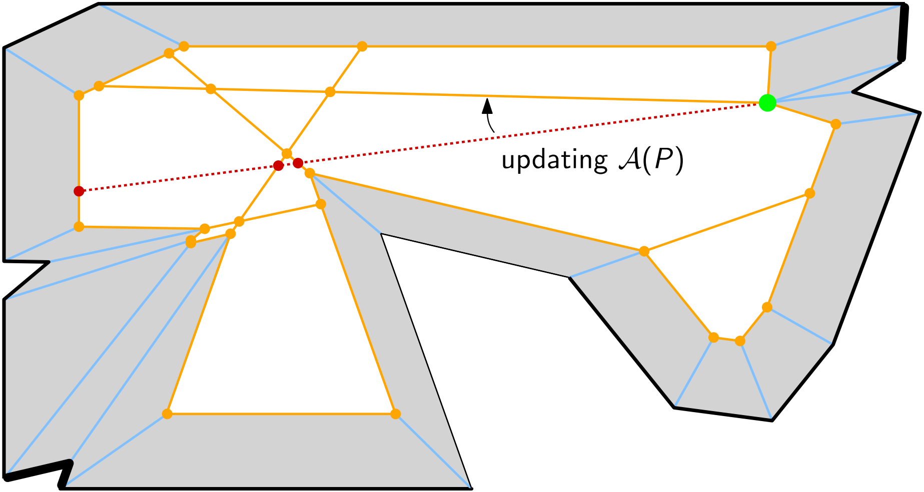

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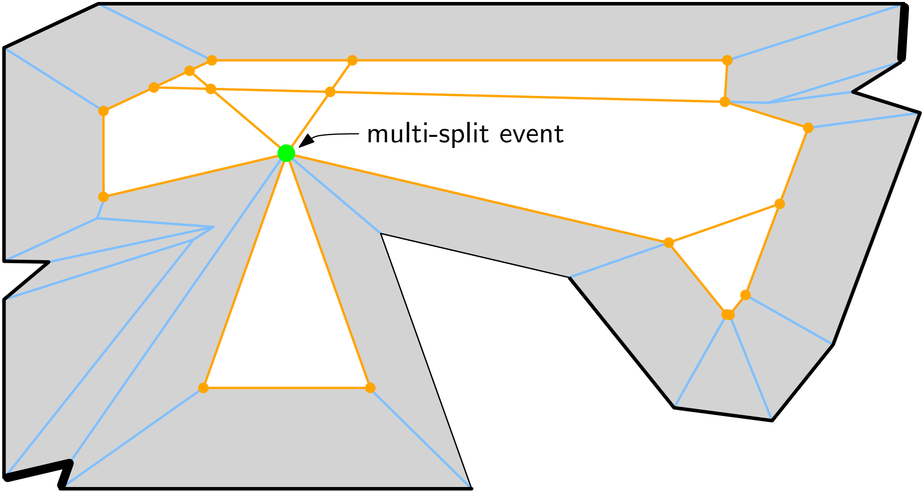

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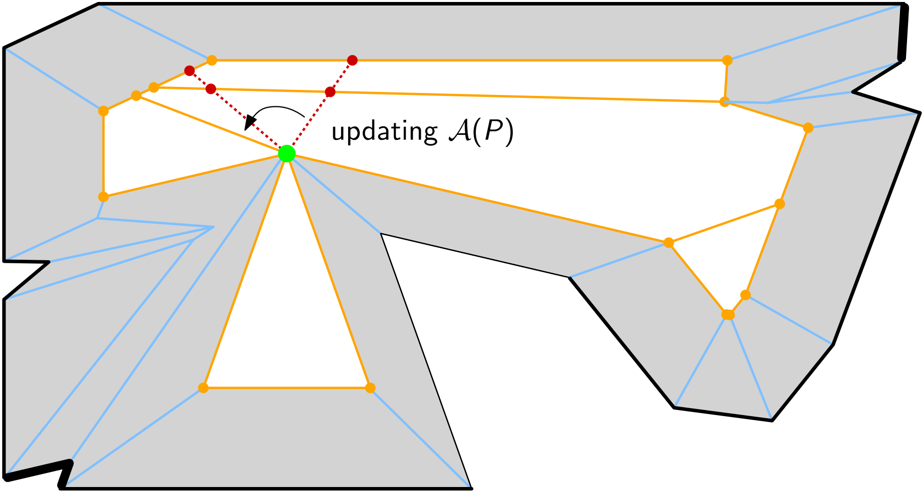

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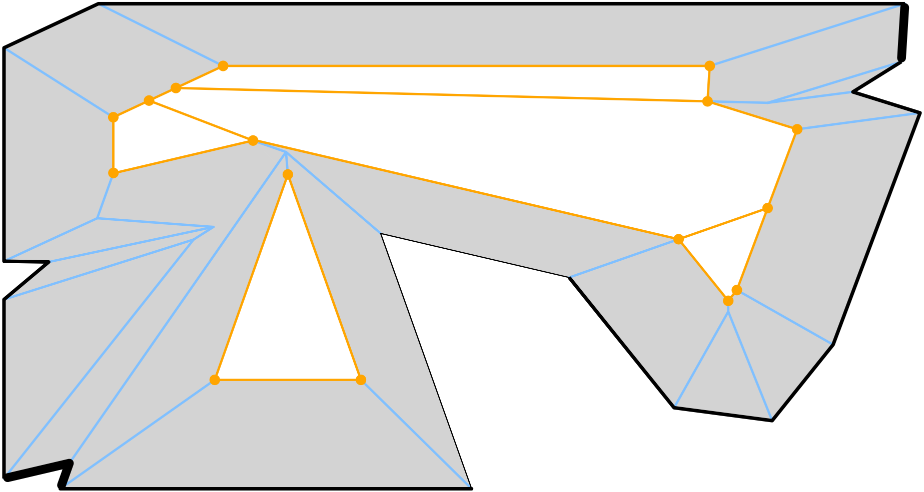

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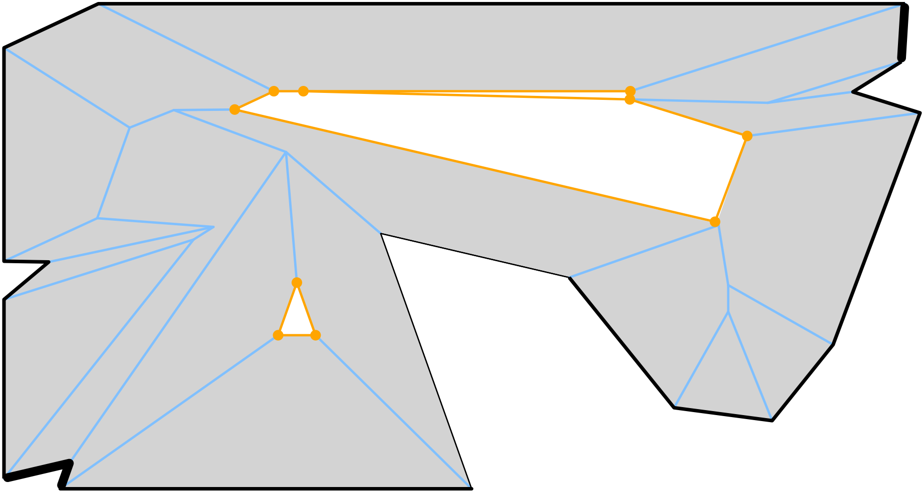

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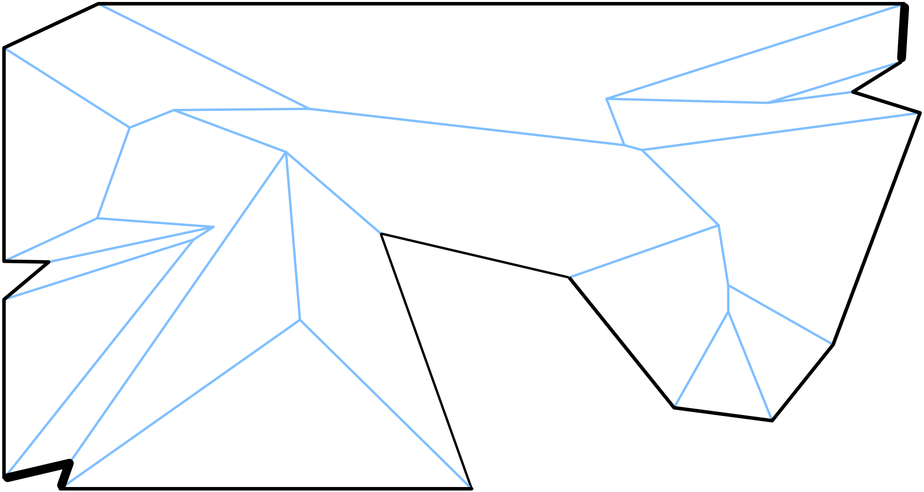

# Complexity Analysis

$P$ consists of $n$ vertices, $r$ of which are reflex. The propagation of $\mathcal{W}_P^*(t, \sigma)$ results in:

- $\mathcal{O}(n)$ split/edge events,
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- Handling one (reflex preserving) edge event takes $\mathcal{O}(n + r + r\log n)$ time:
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### Overall Complexity

| | *time* | *space* |
|---|---|---|
| *classical* | $\mathcal{O}(n^2 + r^3 + nr \log n)$ | $\mathcal{O}(n)$ |

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| | *time* | *space* |
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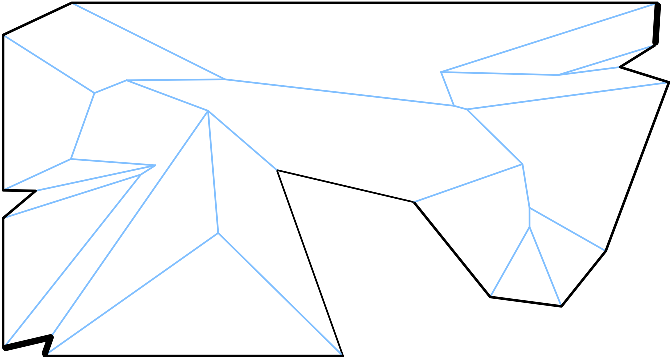
| *trade-off* [5] | $\mathcal{O}(n^2 + r^3/k + nr \log n)$ | $\mathcal{O}(n + kr)$ |

[5] with a fixed $k$ s.t. $1 \leq k \leq r$.

# Q & A



Questions?

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[2] S.-W. Cheng, L. Mencel, and A. Vigneron. A Faster Algorithm for Computing Straight Skeletons. 12(3):44:1–44:21, Apr. 2016.

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